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Review:

Recent progress on the study of distributed economic dispatch in smart grid: an overview*

Guanghui WEN[‡1], Xinghuo YU[2], Zhiwei LIU[3]

[1]*School of Mathematics, Southeast University, Nanjing 211189, China*
[2]*School of Engineering, RMIT University, Melbourne VIC 3000, Australia*
[3]*School of Artificial Intelligence and Automation, Huazhong University of Science and Technology, Wuhan 430074, China*
E-mail: wenguanghui@gmail.com; x.yu@rmit.edu.au; zwliu@hust.edu.cn

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Abstract: Designing an efficient distributed economic dispatch (DED) strategy for the smart grid (SG) in the presence of multiple generators plays a paramount role in obtaining various benefits of a new generation power system, such as easy implementation, low maintenance cost, high energy efficiency, and strong robustness against uncertainties. It has drawn a lot of interest from a wide variety of scientific disciplines, including power engineering, control theory, and applied mathematics. We present a state-of-the-art review of some theoretical advances toward DED in the SG, with a focus on the literature published since 2015. We systematically review the recent results on this topic and subsequently categorize them into distributed discrete- and continuous-time economic dispatches of the SG in the presence of multiple generators. After reviewing the literature, we briefly present some future research directions in DED for the SG, including the distributed security economic dispatch of the SG, distributed fast economic dispatch in the SG with practical constraints, efficient initialization-free DED in the SG, DED in the SG in the presence of smart energy storage batteries and flexible loads, and DED in the SG with artificial intelligence technologies.

Key words: Distributed economic dispatch; Distributed optimization; Smart grid; Continuous-time optimization algorithm; Discrete-time optimization algorithm
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1 Introduction

Electrical grid is one of the most complex infrastructures in modern society. Almost all the essential infrastructures of modern society, ranging from transportation infrastructures to finance infrastructures and to medical infrastructures, are heavily dependent on the reliable supply of electricity. These demands have fostered the emergence of the next-generation infrastructure of electrical power, known as "smart grid" (SG) (Farhangi, 2010), which is a comprehensive upgrade of electrical grid with a range of new information and communication technologies. SG incorporates a large number of distributed energy resources (DERs) (including various gas turbines and wind-power resources), energy storage devices, and flexible loads, leading to great complexity in the energy management of SG. Fig. 1 demonstrates a basic SG structure. Clearly, it incorporates advanced management, control, and communication technologies to improve operational efficiency for power generation units and electricity transmission facilities, and furnish flexible choices for various consumers.

Economic dispatch (ED) is one of the most

‡ Corresponding author
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ORCID: Guanghui WEN, https://orcid.org/0000-0003-0070-8597; Xinghuo YU, https://orcid.org/0000-0001-8093-9787; Zhiwei LIU, https://orcid.org/0000-0003-3005-1792

fundamental and critical issues in energy management of electrical grid. The goal of ED is to schedule power generation of each generator so that the electricity demand in the electrical grid can be supplied entirely in a most economical way, while subject to certain constraints. Practically, short-term load forecasting is the starting point of ED. ED allocates the forecasted load to each generation unit in an economical way. However, as load forecasting may not be accurate, load frequency control is adopted to compensate for the forecasting error using the primary and secondary frequency control. Note that the solution to the ED problem (EDP) corresponds to the set value of the mechanical power output, which is an important parameter in frequency control (Fig. 2). Typically, EDP can be formulated as a constrained optimization problem (COP). To solve this problem, the traditional electrical grid adopts centralized methods and techniques, such as Karush-Kuhn-Tucker (KKT) conditions (Li ZS et al., 2016), interior-point methods (Yan and Quintana, 1997), primal-dual methods (Martinez et al., 2014), quasi-Newton methods (Li ZG et al., 2013), saddle-point methods (Zhang X et al., 2015), and heuristic algorithms (Park et al., 2005). Within the context of the centralized optimization approach, calculations are performed in a centralized computing center that could collect the required global information about the grid. Before a large number of DERs are integrated into the electrical grid, this traditional generation-side dispatching method works well as the number of generators to be scheduled is small. However, it may be inadequate to dispatch generators of an SG with a large number of DERs due to the limitations of the centralized methods, such as low flexibility, low scalability, and high communication/computational cost.

Unlike in the traditional electrical grid, in SG, generators including various DERs will be equipped with sensing, communication, and computing units (Yu XH et al., 2011; Wang YN et al., 2016; Wen et al., 2016). In this context, distributed optimization and computational methods for ED will become feasible and promising. Several benefits, such as easy implementation, low maintenance cost, high energy efficiency, and strong robustness against uncertainties, can be gained using distributed optimization and computational methods (Wang HW et al., 2019). Therefore, distributed ED (DED) has been widely acknowledged. In general, DED aims at designing a distributed algorithm for each generator to dispatch the active power based on the locally available information to cooperatively meet electricity demand at a minimized cost. Many distributed approaches have been developed to deal with the DED problems (DEDPs) under various scenarios, such as communication delay (Yang T et al., 2017), asynchronous setting (Bragin and Luh, 2017), switching communication graph (Yang ZQ et al., 2017), transmission losses (Wang R et al., 2019b), and event-triggered communication (Shi et al., 2020). In this study, we present a state-of-the-art review on some theoretical advances toward DED in SGs. We systematically review the recent results on this topic and subsequently categorize them into distributed discrete- and continuous-time EDs in SG.

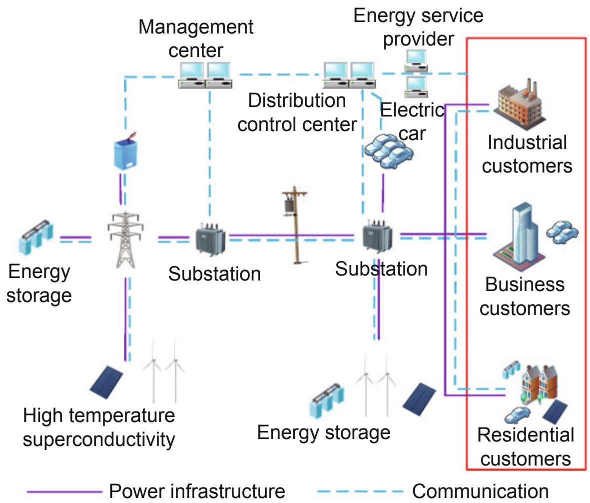

Fig. 1 A simplified framework of smart grid

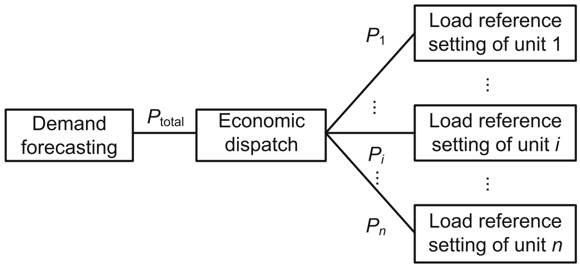

Fig. 2 Execution process of economic dispatch

P_{total} represents the total power demand predicted by demand forecasting

1. Distributed discrete-time algorithm design for ED

Recent results on this topic are categorized into distributed discrete-time algorithm design for EDPs with and without practical operational constraints.

In the case of without operational constraints, the aim is to design distributed discrete-time algorithms to tackle EDP for SG in the presence of the power balance condition. Neglecting the operational constraints on the generators makes it easy for researchers to clarify the effect of communication delays, topological uncertainties, and external disturbances on the distributed algorithms. Compared with designing distributed discrete-time algorithms to solve EDP without operational constraints, the main challenge encountered by considering operational constraints is how to simultaneously deal with the effect of various practical operational constraints, e.g., power losses on transmission lines, ramp rate constraints for generators, and power balance constraints on EDPs.

2. Distributed continuous-time algorithm design for ED

The existing research advances on such a topic are categorized into distributed continuous-time algorithm design for ED with various generators' constraints and distributed continuous-time algorithm design for ED with communication imperfections. Compared with the distributed discrete-time ED algorithms, the distinguishing feature of the continuous-time ED algorithms lies in that they can avoid the difficulty encountered by step size selections for convergence analysis and thus can facilitate theoretical analysis.

2 Preliminaries

First, we present some basic concepts and useful lemmas in graph theory, nonsmooth analysis, and discrete-time/continuous-time consensus algorithms in multi-agent systems.

The communication network of generation units in SG can be modeled by a directed graph (digraph) or an undirected graph $\mathcal{G}(\mathcal{V},\mathcal{E})$, where $\mathcal{V}$ denotes the set of generators (nodes) and $\mathcal{E}$ denotes the set of communication links (edges) between generators. For a digraph, we say that i can receive information from j if $(j,i)\in\mathcal{E}$; let $\mathcal{N}_i^+=\{j|(j,i)\in\mathcal{E}\}$ and $\mathcal{N}_i^-=\{j|(i,j)\in\mathcal{E}\}$ denote the in- and out-neighbor of the i^{th} generator, respectively. For an undirected graph, i and j can receive each other's information if $(i,j)\in\mathcal{E}$. $\mathcal{N}_i$ denotes the neighbor set of i. $\mathcal{G}$ is a strongly connected digraph if there is a directed path from i to j for any two distinct nodes $i,j\in\mathcal{V}$. Correspondingly, $\mathcal{G}$ is a connected undirected graph if there is an undirected path between any two distinct nodes $i,j\in\mathcal{V}$. A weighted digraph $\mathcal{G}=(\mathcal{V},\mathcal{E},\boldsymbol{A})$ consists of a digraph $(\mathcal{V},\mathcal{E})$ and an adjacency matrix $\boldsymbol{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$ with $a_{ij}>0$ if and only if $(j,i)\in\mathcal{E}$. The weighted out-degree and in-degree of node i are $d_{\text{out}}(i)=\sum_{j=1}^N a_{ji}$ and $d_{\text{in}}(i)=\sum_{j=1}^N a_{ij}$, respectively. The Laplacian matrix is given by $\boldsymbol{L}=\boldsymbol{D}_{\text{in}}-\boldsymbol{A}$, where $\boldsymbol{D}_{\text{in}}$ is the diagonal matrix with its i^{th} diagonal element equal to $d_{\text{in}}(i)$ for $i=1,2,\ldots,N$. If $\mathcal{G}$ is strongly connected, then $\mathbf{0}$ is a simple eigenvalue of $\boldsymbol{L}$. We say that $\mathcal{G}$ is weight-balanced if $d_{\text{out}}(i)=d_{\text{in}}(i)$ for all $i\in\mathcal{V}$. Under the condition that $\mathcal{G}$ is weight-balanced, we may obtain $\mathbf{1}_N^{\mathrm{T}}\boldsymbol{L}=\mathbf{0}$ and $\boldsymbol{L}_{\text{s}}=(\boldsymbol{L}+\boldsymbol{L}^{\mathrm{T}})/2\geq 0$, where $\mathbf{1}_N$ is an N-dimensional column vector with each element being 1. Notably, an arbitrarily undirected graph is weight-balanced. If $\mathcal{G}$ is weight-balanced and strongly connected, then 0 is a simple eigenvalue of $\boldsymbol{L}_{\text{s}}$ and for all $\boldsymbol{x}\in\mathbb{R}^N$, we have $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{L}_{\text{s}}\boldsymbol{x}\geq\lambda_2(\boldsymbol{L}_{\text{s}})\left\|\boldsymbol{x}-\dfrac{1}{N}(\mathbf{1}_N^{\mathrm{T}}\boldsymbol{x})\mathbf{1}_N\right\|^2$, where $\lambda_2(\boldsymbol{L}_{\text{s}})$ is the smallest nonzero eigenvalue of $\boldsymbol{L}_{\text{s}}$.

Function $f:\mathbb{R}^n\to\mathbb{R}^m$ is a locally Lipschitz function at $\boldsymbol{x}\in\mathbb{R}^n$ if $\varepsilon>0$, and a positive scalar $L_{\boldsymbol{x}}$ exists such that $\|f(\boldsymbol{x}_1)-f(\boldsymbol{x}_2)\|\leq L_{\boldsymbol{x}}\|\boldsymbol{x}_1-\boldsymbol{x}_2\|$ for $\boldsymbol{x}_1,\boldsymbol{x}_2\in B(\boldsymbol{x},\varepsilon)$, where $\|\cdot\|$ is the Euclidean norm and $B(\boldsymbol{x},\varepsilon)$ is a Euclidean ball with radius ε and center $\boldsymbol{x}$. Given a locally Lipschitz function f, let $\varOmega_f$ be the set of points (of measure zero), where f is non-differentiable, and then the generalized gradient ∂f is defined as $\partial f=\text{co}\{\lim_{t\to\infty}\nabla f(\boldsymbol{x}_i)|\boldsymbol{x}_i\to\boldsymbol{x},\ \boldsymbol{x}_i\notin S\cup\varOmega_f\}$, where "co" represents the convex hull and S is an arbitrary set with measure zero. For differential inclusion $\dot{\boldsymbol{x}}\in\mathcal{K}(\boldsymbol{x})$, where $\mathcal{K}:\mathbb{R}^n\mapsto\mathbb{R}^n$ is a set-valued map, its solution on $[0,T]\subset\mathbb{R}$ is an absolutely continuous map $\boldsymbol{x}:[0,T]\to\mathbb{R}^n$ that satisfies the differential inclusion almost everywhere. If $\mathcal{K}$ is locally bounded, i.e., upper semicontinuous, and has nonempty, compact, and convex values, then the existence of a solution can be guaranteed. The corresponding LaSalle invariance principle can be found in Cortés (2008).

Matrix $\boldsymbol{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$ is row-stochastic if $\sum_{j=1}^N a_{ij}=1$ for any $i\in\{1,2,\ldots,N\}$. Matrix $\boldsymbol{A}$ is column-stochastic if $\sum_{i=1}^N a_{ij}=1$ for any $j\in\{1,2,\ldots,N\}$. The classical distributed discrete-time (first-order) consensus algorithm over a fixed interaction graph can be represented as follows (Ren et al., 2007):

$$\boldsymbol{x}_i[k+1] = \sum_{j=1}^{N} a_{ij}\boldsymbol{x}_j[k], \tag{1}$$

where $\boldsymbol{x}_j[k] \in \mathbb{R}^n$ is the state of node j at step k (representing the k^{th} communication event) and a_{ij} is the $(i,j)^{\text{th}}$-entry of the adjacency matrix $\boldsymbol{A}$ associated with an interaction graph. It is generally assumed that matrix $\boldsymbol{A}$ in Eq. (1) is a row-stochastic matrix with $a_{ii} > 0$. Consensus is achieved in a network if $\lim_{k\to+\infty}\|\boldsymbol{x}_i(k) - \boldsymbol{x}_j(k)\| = 0$ for any $i,j \in \{1,2,\ldots,N\}$.

The classical distributed continuous-time consensus protocol over a fixed interaction graph can be described as follows (Ren et al., 2007):

$$\dot{\boldsymbol{x}}_i(t) = c\sum_{j=1}^{N} a_{ij}[\boldsymbol{x}_j(t) - \boldsymbol{x}_i(t)], \tag{2}$$

where $\boldsymbol{x}_i(t) \in \mathbb{R}^n$ is the state of agent i at time t, $c > 0$ the coupling strength, and $\boldsymbol{A} = [a_{ij}]_{N\times N}$ the adjacency matrix of the interaction graph. It is generally assumed that each diagonal entry of matrix $\boldsymbol{A}$ in system (2) is equal to 0. Thus, system (2) can be equivalently written as $\dot{\boldsymbol{x}}_i(t) = -c\sum_{j=1}^{N} l_{ij}\boldsymbol{x}_j(t)$ with $\boldsymbol{L} = [l_{ij}]_{N\times N}$ being the corresponding Laplacian matrix. Since each agent uses only its neighbors' information, the protocol is in a distributed fashion. We say that the consensus is achieved if $\lim_{t\to+\infty}\|\boldsymbol{x}_i(t) - \boldsymbol{x}_j(t)\| = 0$ for $i,j = 1,2,\ldots,N$ and for any given initial condition $\boldsymbol{x}_k(0)$ ($k = 1,2,\ldots,N$).

3 Distributed discrete-time economic dispatch in smart grid

Consider a traditional EDP with a supply-demand balance condition and generator constraints in SG, which is formulated as

$$\begin{aligned} &\min \sum_{i=1}^{N} C_i(P_i) \\ &\text{s.t.} \sum_{i=1}^{N} P_i = P_{\text{total}},\ P_i^{\min} \le P_i \le P_i^{\max}, \end{aligned} \tag{3}$$

where P_i is the power generation of the i^{th} generator, $C_i(P_i)$ is the corresponding generation cost, P_{total} is the total power demand, and $P_i^{\min}$ and $P_i^{\max}$ limit the capacity of the i^{th} generator.

The main objective of ED in SG is to make the power system run under most economical conditions while meeting the power demand and generator capacity constraints. Practically, some operational constraints in the SG, such as transmission losses, prohibited operating zones, ramp rate constraints, and valve-point loadings, should be considered. To deal with EDP, various centralized ED methods have been suggested. However, the previous centralized ED methods rely on some global information of the grid and sometimes need detailed information of each generator, which may be inapplicable to SGs in the presence of a large number of DERs. Therefore, distributed algorithms are more suitable for addressing EDP of the SG with a lot of DERs due to their robustness, low information requirements, and excellent scalability. In this section, we will give an overview of the research progress of the distributed discrete-time ED in SG in recent years. We generally focus on the following two aspects: simple ED without practical operational constraints and ED with transmission line losses, unit ramp rate constraints, prohibited operating zones, and valve-point loading effects.

3.1 Distributed discrete-time economic dispatch without a practical operational constraint

The cost of each generator is usually modeled by a quadratic function which is a typical convex function. For EDP (3) with power demand constraints and generator constraints, when the distributed generators in the SG operate in an optimal state, the incremental costs of all generators have a common value. Consequently, the concept of taking incremental costs of the generators as a consensus variable emerges, and consensus protocols are employed to solve EDP in the SG. By feeding the mismatch between power generation and demand back to the consensus algorithm, the demand and supply balance constraint in EDP is guaranteed.

Zhang Z and Chow (2011, 2012) proposed several new kinds of decentralized methods to solve EDP by the first-order consensus technique, where the incremental cost of each generator was selected as the consensus variable. Specifically, the relationship between communication topology and convergence speed of the EDP algorithm was analyzed in Zhang Z and Chow (2012). The algorithms used in Zhang

Z and Chow (2011, 2012) are given as follows:

$$\lambda_i[k+1] = \sum_{j=1}^{N} d_{ij}\lambda_j[k], \tag{4a}$$

$$\lambda_{\text{leader}}[k+1] = \sum_{j=1}^{N} d_{ij}\lambda_j[k] + \varepsilon \Delta P, \tag{4b}$$

where $\lambda_i[k]$ is the consensus variable of the i^{th} generator at step k, $\lambda_{\text{leader}}[k]$ is a variable associated with the leading generator, d_{ij} satisfies $\sum_{j=1}^{N} d_{ij} = 1$, ΔP is the total deviation of power supply and demand, and ε is a positive scalar. The leading generator updates its consensus variable according to Eq. (4b), and other generators will update their consensus variables according to Eq. (4a). Here, the leading generator is responsible for gathering the deviation of power supply and demand. Dominguez-Garcia et al. (2012) developed a decentralized ratio consensus algorithm to achieve optimal dispatch of DER, and introduced an external leading generator to comprehend the total demand. In the absence of a central controller or a leading generator, how to obtain power mismatches in a fully distributed way and how to meet the supply-demand balance constraint are two challenges for adopting a distributed consensus-based approach to solve EDP in the SG.

Taking incremental cost as a consensus variable, many distributed consensus-based discrete-time ED algorithms were proposed (Kar and Hug, 2012; Binetti et al., 2013; Yang SP et al., 2013; Yang T et al., 2016; Yang ZQ et al., 2017; Zhao CC et al., 2017a; Li Q et al., 2019; Wang R et al., 2019a). In the distributed discrete-time algorithms for EDPs in the SG, a common update rule of the local mismatch estimation (Yang SP et al., 2013; Yang T et al., 2016; Wang R et al., 2019a) is given by

$$\Delta P_i[k+1] = \sum_{j\in\mathcal{N}_i} d_{ij}\Delta P_j[k] - (P_i[k+1]-P_i[k]), \tag{5}$$

where $\Delta P_i[k]$ and $P_i[k]$ are the local mismatch estimate and generation of the i^{th} generator, respectively. Using the local mismatch estimate to adjust the power outputs of generators and coordinating with their local neighbors, incremental costs of all generators converge to a consensus value. Moreover, it realizes the objective of supply-demand balance without a leading generator collecting the power output of each generator.

Results based on the equal incremental cost criterion for EDP on undirected communication networks have been reported (Kar and Hug, 2012; Yang ZQ et al., 2017; Li Q et al., 2019; Wang R et al., 2019a). Kar and Hug (2012) introduced an innovation item based on the consensus item, used the consensus item to make the incremental cost converge to a common value, and used the innovation item to ensure that the power generation and demand are equal. The proposed distributed algorithm for EDP in Yang ZQ et al. (2017) is suitable for time-varying power demand and switching communication topology. Wang R et al. (2019a) proposed a consensus-based distributed algorithm, where the gain of mismatched feedback is time-varying. Li Q et al. (2019) embedded the proportional integral (PI) frequency controller and neural network controller in the consensus protocol to generate a DED algorithm, which could realize real-time dispatch of power systems instead of traditional periodic dispatch. Li Q et al. (2019) also considered the problems of communication failure and line loss.

Considering the possible packet loss, communication failure, and communication asymmetry, the bidirectional communication on undirected communication topologies is restrictive. Distributed discrete-time EDP on directed communication graphs has been further studied (Binetti et al., 2013; Yang SP et al., 2013; Xing et al., 2015b; Yang T et al., 2016). Xing et al. (2015b) proposed a fully distributed consensus-based bisection approach to solve EDP with strongly connected digraphs; this algorithm is suitable for the case where the cost function is generally convex and not limited to be quadratic. Yang SP et al. (2013) employed the same incremental cost criterion and proposed an algorithm that ensures the incremental costs converging to the optimal value under the strong connectivity assumption of the communication topology; the proposed algorithm can be regarded as a distributed implementation of the standard λ-iteration method. Later, based on Yang SP et al. (2013), Yang T et al. (2016) proposed a distributed algorithm using the minimum time steps for EDP with digraphs, which could speed up convergence while reducing communication costs.

In addition to the incremental cost consensus approach, distributed gradient-based discrete-time algorithms have been employed to solve EDP in the SG (Li CJ et al., 2014, 2016; Zhang W et al., 2015;

Guo et al., 2016). The concept of the DED gradient-based algorithm is that within the capacity of generators, the update rule of power generation of each generator is

$$P_i[k+1] = P_i[k] - \alpha[k] \sum_{j=1}^{N} w_{ij}[k] \nabla C_j(P_j[k]), \quad (6)$$

where $P_i[k]$ is the generation of the i^{th} generator at the k^{th} iteration, $\alpha[k]$ the step size, $\nabla C_i(P_i[k])$ the gradient of the cost function $C_i(P_i)$ at $P_i[k]$, and $w_{ij}[k]$ the $(i,j)^{\text{th}}$-entry of matrix $\boldsymbol{W}[k]$ satisfying $\boldsymbol{1}_N^{\mathrm{T}}\boldsymbol{W}[k] = \boldsymbol{0}$ and $\boldsymbol{W}[k]\boldsymbol{1}_N = \boldsymbol{0}$. The distributed generators cooperatively minimize the global objective function by exchanging gradient information of the local cost functions with their neighbors. To make the generators operate within their capacity in the process of ED, Li CJ et al. (2014) used a θ-logarithmic barrier, and Zhang W et al. (2015) exploited the projection method to ensure that the power outputs of all generators were within their capacity range. Specifically, in Li CJ et al. (2016), the event-triggered mechanism and the gradient-based method were combined to reduce the communication cost while ensuring the convergence speed of the algorithm. As mentioned in Guo et al. (2016), for the DED gradient-based algorithms, the choice of step size and the allocation of power demand at the initial time are vital.

Notably, most of the distributed discrete-time algorithms designed for EDP in the SG are based on an ideal communication environment. A perfect communication condition is the premise of ensuring the performance of the proposed algorithms. Therefore, the quality of the communication service has become an essential consideration of economic scheduling in the SG. In practical applications, the limited bandwidth of communication channels and the delay of data transmission will decrease the performance of ED algorithms. Therefore, in the study of the EDP in the SG, researchers have made considerable effort to analyze the effect of communication packet loss and communication delay on the achievement of ED. For example, considering communication losses in communication networks, a robust distributed algorithm based on the equal incremental cost criterion for EDP was introduced in Zhang Y et al. (2016), and its convergence could be guaranteed by connecting communication topology even if there were communication losses. Based on the push-sum and gradient approaches, Wu et al. (2017) studied EDP on digraphs with communication losses. Duan and Chow (2020) proposed a robust consensus-based distributed discrete-time algorithm for EDP using a correction method to compensate for the packet loss. For EDP with time delays, Yang T et al. (2015) studied the influence of time delays on the distributed algorithm, where time delays were constant. Considering the more general nonuniform delays in actual situations, Yang T et al. (2017) studied EDP over time-varying communication networks with nonuniform delays using the gradient push-sum method. Yuan et al. (2019) developed a distributed asynchronous algorithm based on Yang T et al. (2017).

No matter the incremental cost consensus algorithm or the gradient-based method, the information that the distributed generators need to exchange, such as local incremental cost estimate, gradient information of cost functions, and power outputs, involves the privacy information of generators. With the development of SG, privacy protection and security are becoming increasingly important. From the perspective of privacy protection, Pourbabak et al. (2018) developed a novel distributed algorithm for EDP, in which the information that the generator needs to exchange with its neighbors was only the estimate of the local mismatch between the generating capacity and demand, and did not disclose private information like the incremental cost and local cost function. In addition to privacy protection, cyber-attacks are one of the significant threats to SG. The existence of cyber-attacks, such as malicious nodes that inject false data into the transmitted information, may paralyze the whole power system, making power scheduling impossible. Therefore, it is vital to design attack-resistant distributed discrete-time algorithms for EDP in the SG. Some research advances on ED in the presence of cyber-attacks can be found (Liu et al., 2016; Zeng et al., 2017; Zhao CC et al., 2017a; Li PK et al., 2018).

3.2 Distributed discrete-time economic dispatch with practical operational constraints

To simplify the analysis, some of the studies on typical EDP in the SG consider only the generator capacity and supply-demand balance constraints. However, in the actual operation and control of the power system, there are often various practical constraints, such as power losses on the transmission lines, ramp rate limits for power generators because the power output cannot change in time, and prohibited operating zones where the generator load is not stable. These practical operational constraints make EDPs have non-convex characteristics, and hence their solutions are more complex.

Considering power losses in EDP, the supply-demand balance constraint becomes

$$\sum_{i=1}^{N} P_i = P_{\text{total}} + P_{\text{loss}}, \tag{7}$$

where P_{loss} is the transmission loss. P_{loss} is usually expressed as a simple quadratic function (Binetti et al., 2014b; Kouveliotis-Lysikatos and Hatziargyriou, 2017; Zhao CC et al., 2017b; Wang R et al., 2019b), and the equality constraint with such transmission loss makes EDPs become non-convex optimization problems. Binetti et al. (2014b) proposed a distributed consensus-based algorithm for EDP with transmission losses, and considered power loss of each transmission line. Zhang Y and Chow (2015) studied EDP with transmission losses employing the primal-dual method, where each bus communicated with its neighbors for local mismatch estimate. Kouveliotis-Lysikatos and Hatziargyriou (2017) used Kron's formula to calculate power losses and applied a replication dynamics model to solve EDP. Considering the transmission losses with matrix quadratic form, Xing et al. (2015a) designed a distributed average consensus-based algorithm. Xing et al. (2015a) used the local dichotomy to calculate the optimal incremental cost, but required a leading generator to comprehend the total power demand of the generators. Using the ratio consensus, Zhao CC et al. (2017b) studied the energy management with transmission losses on the directed communication topology of the SG. By transforming the equality constraint with transmission losses into the inequality constraint by the relaxation method, Zhao CC et al. (2017b) transformed a non-convex problem into a convex problem, where some sufficient conditions for equivalence of the two problems were further provided. A distributed asynchronous algorithm can be found in Wang R et al. (2019b) to deal with EDP with transmission losses.

In the distributed discrete-time ED algorithm, the ramp rate limits can be described as

$$-P_i^{\text{ramp\_1}} \le P_i[k] - P_i[k-1] \le P_i^{\text{ramp\_2}}, \tag{8}$$

where $P_i^{\text{ramp\_1}}$ and $P_i^{\text{ramp\_2}}$ are the generators' ramp rate limits which provide allowable tuning of generator i's output power within two timing intervals. To make the generators run stably, the prohibited operating zones in EDP are as follows:

$$\begin{cases} P_i^{\min} \le P_i \le P_{i,1}, \\ P_{i,(l-1)} \le P_i \le P_{i,l}, \\ P_{i,m_i} \le P_i \le P_i^{\max}, \end{cases} \tag{9}$$

where $l = 2, 3, \ldots, m_i$, and m_i is the number of prohibited zones for generator i. For the valve-point loading effects, the cost functions of generators are usually approximated by adding the sine function after the quadratic function (Binetti et al., 2014a; Xie et al., 2018; Li FY et al., 2019). Some more distributed ED algorithms under various operational constraints, like ramp rate limits, prohibited operating zones, and valve-point loading effects, have been designed (Binetti et al., 2014a; Xu et al., 2015; Xie et al., 2018; Zhou et al., 2018; Li FY et al., 2019). Binetti et al. (2014a) designed a distributed algorithm based on auction technology and the consensus algorithm to solve non-convex EDP in the SG, including valve-point loading effects, multi-fuel selection, prohibited operating zones, and other constraints. Considering the generator capacity, supply-demand balance constraint, and ramp rate constraints, Xu et al. (2015) transformed the discrete-time EDP into a knapsack problem and designed a DED algorithm using the notion of dynamic programming. Zhou et al. (2018) reported a distributed consensus-based EDP with ramp rate limits. Xie et al. (2018) proposed a distributed stochastic gradient-free method to solve non-convex EDP with practical operational constraints. Considering the valve-point loading effects, Li FY et al. (2019) studied EDP on strongly connected digraphs using the distributed pattern search algorithm.

Compared with EDP without practical operational constraints, research results of the distributed discrete-time ED algorithms with practical operational constraints in the SG are relatively few. To improve the completeness of solving EDP, the distributed algorithm of non-convex EDP with various practical operational constraints needs further exploration.

4 Distributed continuous-time economic dispatch in smart grid

In this section, we elaborately discuss the typical distributed continuous-time ED algorithms in the SG.

In recent years, as discussed in the previous section, there have been many interesting works on distributed discrete-time algorithms to tackle economic power dispatch problems. Continuous-time ED algorithms, or broadly saying, distributed optimization ones, have rapidly developed in recent years mainly due to the following reasons and consideration: With the development of cyber-physical techniques and application scenarios of real-time optimization techniques, it is vital to pay much more attention to continuous-time settings since numerous practical industrial systems operate in a continuous-time manner. On one hand, continuous-time algorithms, especially neuro-dynamic optimization ones, have attracted considerable attention with the well-developed and powerful control techniques and the easy realization of hardware implementations. Multiple types of continuous-time algorithms have been designed to solve various linear and nonlinear programming problems. Among these applications, various distributed continuous-time algorithms have been designed for distributed optimization of multi-agent networks (Yang T et al., 2019). On the other hand, from the viewpoint of continuous-time dynamics, more reliable and rigorous theoretical results are expected to be obtained via well-established analytic tools and theory, such as Lyapunov stability theory and nonsmooth analysis.

In what follows, we will summarize the distributed continuous-time ED algorithms with categories regarding the various generators' constraints and under various communication imperfections.

4.1 Distributed continuous-time economic dispatch with generators' constraints

According to whether the individual generators subject to constraints, the study of distributed continuous-time EDPs can be classified roughly into distributed continuous-time EDPs without any constraint on generators (Mudumbai et al., 2012; Li KX et al., 2018; Zhao ZY and Chen, 2018; Li H et al., 2019) and distributed continuous-time EDPs with generators subject to constraints. Furthermore, the study of distributed continuous-time EDPs with generators subject to constraints can be categorized as distributed continuous-time EDPs with generators subject to box-constraints (each generator has upper and lower bounds on the power it could produce) (Cherukuri and Cortés, 2015, 2016; Chen et al., 2017; Chen and Zhao, 2018, 2020; He et al., 2018, 2019; Li CJ et al., 2018; Yu WW et al., 2018; Bai et al., 2019; Huang et al., 2019; Mao et al., 2019; Wang D et al., 2019; Dai et al., 2020) and distributed continuous-time EDPs with generators subject to non-box constraints (Yi et al., 2016; Liang et al., 2019; Zhu et al., 2019).

Without considering any box-constraint for generators, Mudumbai et al. (2012) designed a new kind of distributed algorithm for frequency control and optimal ED with heterogeneous power generators. Specifically, it was assumed in Mudumbai et al. (2012) that the cost function for each generator was twice differentiable and strictly convex, and that the total power imbalance was available in algorithm design. Li KX et al. (2018) considered a kind of nonsmooth cost function for ED, where the concept of dual agents whose roles are to communicate with other agents on behalf of primal agents was presented.

Cherukuri and Cortés (2015) considered EDP with a weight-balanced communication digraph and generators subject to box-constraints. Based on the nonsmooth analysis and stability theory of differential inclusions, a class of algorithms with distributed Laplacian-gradient dynamics were designed to guarantee that the considered EDP could be solved asymptotically. Yu WW et al. (2018) proposed a framework for economic power dispatch based on distributed optimization and consensus dynamics. They introduced two fully distributed protocols to realize the consensus of the incremental costs among generators with or without box-like capacity constraints. It was proved in Yu WW et al. (2018) that if the modified incremental costs of all generators reach consensus, the power output of each generator would achieve the corresponding optimal solution. Since the convergence speed of the first-order algorithm is relatively low, it is desirable to accelerate the convergence by developing algorithms with high-order dynamics. Within this context, distributed second-order algorithms were further studied in He et al. (2018) and Wang D et al. (2019).

Particularly, He et al. (2018) proposed two kinds of second-order continuous-time algorithms to solve the economic power dispatch problem. By applying the nonsmooth exact penalty function, the following differential equation was established and analyzed in He et al. (2018):

$$\begin{cases} \dot{P}_i(t) = -\sum_{j=1, j\neq i}^{N} a_{ij}[Q_j(t) - Q_i(t)], \\ \dot{Q}_i(t) \in -\varepsilon Q_i(t) - \partial f_i^{\theta}(P_i(t)), \end{cases} \tag{10}$$

where $i = 1, 2, \ldots, N$.

Wang D et al. (2019) proposed a second-order continuous-time algorithm to seek the optimal solution to DEDPs over connected communication graphs based on the saddle-point dynamics and differential inclusion theory. The obtained algorithms can be further extended to the case with switching communication topologies. Dai et al. (2020) developed a distributed algorithm for ED with unknown cost functions. The state-action-value function approximation using semi-gradient Q-learning and the multiplier splitting technique for distributed optimization were combined to obtain the optimal solution in Dai et al. (2020).

Another crucial concern for ED in the SG is the initialization procedures for all generators. Many existing results require meticulous initialization coordination to ensure that the supply-demand balance can be met at the initial dispatch instants. However, for dynamic ED, the initialization coordination has been re-performed whenever generation cost changes or local load demand varies, or some distributed generators are added to or removed from the SG. With the penetration of ever-increasing renewable energy and aiming at highly flexible SGs, this issue should be well emphasized and initialization-free DED algorithms are needed. Cherukuri and Cortés (2016) considered initialization-free ED algorithms over a strongly connected, weight-balanced digraph. It has been proved that the dynamics of power mismatch between supply and demand is input-to-state stable, which indicates that the algorithm would be robust to various initialization errors. Combined with saddle-point dynamics and consensus algorithms, Bai et al. (2019) designed a distributed algorithm for ED on undirected graphs with a mild requirement for the initial conditions. The consensus protocols were applied to distributively estimate the global information, and the saddle-point dynamics aims to search for the optimal solution. Using the singular perturbation theory, exponential stability (practical stability) of the optimal solution was derived without (with) the capacity limits.

Owing to the high requirements from industrial applications in SGs, such as security operations of inverter-based devices, traditional ED algorithms considering only box-like constraints are beyond adequate. It is important but also rather challenging to develop distributed algorithms to deal with more general capacity constraints, since the KKT conditions for optimization problems with and without general local constraints are dramatically different. Yi et al. (2016) addressed the distributed optimal resource allocation problem with multi-dimensional decision variables and non-box local feasibility constraints, where the designed algorithms can be applied to solve dynamic EDP. Liang et al. (2019) developed a kind of distributed algorithm to solve the resource allocation problems with local set constraints and coupled inequality constraints, where the underlying network topology was modeled by a weight-balanced graph. In practice, the sub-optimal solution is sometimes desirable due to the consideration for low computational cost and simple algorithm design procedure. Within the context of distributed sub-optimal resource allocation, some new analysis methodologies for sub-optimality and convergence of the distributed algorithms were derived in Liang et al. (2019), where the twice differentiability condition of the cost function was removed. Zhu et al. (2019) designed a continuous-time projection algorithm for resource allocation problems (which are closely related to EDPs) with local convex sets over a strongly connected and weight-balanced digraph. The algorithm suggested in Zhu et al. (2019) does not require pre-computation on the tangent cone, and is also privacy-preserving. Using tools from nonsmooth analysis, the asymptotic convergence of the proposed algorithm to the optimal solution was proved in Zhu et al. (2019).

In the investigation of DEDPs, the convergence speed of the algorithm plays a significant role, since the renewable energy resources penetrate deeply into the SG and the generation and loads usually vary frequently. Finite-time convergence is preferable in practical applications when the DED algorithms are usually viewed as periodic optimization procedures.

Different from the DED algorithms mentioned above, where the asymptotic convergence speed is ensured, it is necessary to further investigate the algorithms with prescribed finite-time convergence speed. Chen et al. (2017) proposed a finite-time DED algorithm with a sign function term and an integral term to calculate scheduled active power, considering both supply-demand balance and generation capacity constraints. The algorithm guarantees the finite-time convergence for the optimal solution under undirected and connected graphs. In many related works on ED, the gradients or sub-gradients of local cost functions are assumed to be bounded or have a linear-increasing assumption. Mao et al. (2019) further applied the sign function term to relax such restrictions, and the finite-time convergence for more general convex objective functions was derived based on the finite-time control theory. The main benefit of finite-time ED algorithms is that they have higher convergence speed. Notably, the settling time of finite-time algorithms is directly dependent on the initial state of the whole network system. However, for practical SGs, the initial states of generators are costly to obtain and may vary a lot in each dispatch period. It is more advantageous to design a new distributed algorithm with prescribed settling time, which is independent of the initial states of generators. This falls into the investigation of fixed-time control. Within this context, Li H et al. (2019) proposed a DED algorithm with fixed-time convergence without box-constraints but subject to supply-demand balance constraints. For the case with more general generators' capacity constraints, designing distributed initialization-free continuous-time ED algorithms is still facing challenges, and it is worth devoting much effort to this topic in the future.

4.2 Distributed continuous-time economic dispatch with communication imperfections

One of the most common factors considered within the framework of the distributed continuous-time ED subject to communication imperfections is communication delay. Note that time delay is ubiquitous in almost all practical systems since it is a basic property that exists in the processes of sensing (a certain amount of time needed for the sensor to acquire information), communication (limited communication speed within the communication network), control (execution time needed to implement control inputs), and computation (computation time to generate allowable control inputs). In various distributed continuous-time ED algorithms, the communication network was assumed to be perfect, and no communication time delay was involved. However, when applying the distributed algorithms in real application scenarios, the communication time delays are inevitable, which might degrade the convergence speed or even destroy the convergence of the algorithms. It is thus vital to further investigate the distributed continuous-time algorithms by fully considering the effect of time delays. The main research concern for this topic is to study under what conditions regarding the time delays the effectiveness of algorithms can still be assured. Or equivalently, what is the effect of communication time delays on the distributed continuous-time ED algorithms; also, will one be able to find the conditions of time delays such that the optimization problem can be solved?

Zhu et al. (2016) proposed a modified distributed algorithm for the evolution of incremental cost as

$$\dot{y}_i(t) = -2c\gamma_i \sum_{j=1}^{N} l_{ij} y_j(t-\tau_{ij}) \tag{11}$$

for generators $i = 1, 2, \ldots, N$, where $c > 0$ represents the coupling strength among neighbor generators, $\gamma_i > 0$ the parameter related to the cost function of generator i, and $\tau_{ij} > 0$ the time needed for the information transformation from generator i to generator j. By taking the Laplace transformation on the variable $y_i(t)$ and analyzing the zeros of the transfer function, a necessary and sufficient condition for the allowable time delay upper bound was derived in Zhu et al. (2016). Considering the box-constraints for generators, Chen and Zhao (2018) suggested a second-order distributed continuous-time ED algorithm, where heterogenous time delays were included in the algorithm. Using the generalized Nyquist criterion, the impacts of time delays on dynamic performance were extensively analyzed, and a sufficient condition for the maximum allowable delay bounds to ensure DED was derived. Under a strongly connected and weight-balanced communication digraph, Zhao ZY and Chen (2018) designed an event-based DED algorithm, where constant communication delay was considered. Sufficient conditions regarding the event-triggered parameters and the constant delay were further presented in Zhao ZY and

Chen (2018), under which the considered ED can be achieved. Considering the generation capacity constraints, Chen and Zhao (2020) proposed a kind of distributed second-order algorithm with nonuniform communication delays and self-delays. The impacts of delays on the convergence of the proposed algorithm were analyzed using the matrix perturbation analysis method and the generalized Nyquist criterion, and the strict upper bound of time delays on the convergence of algorithms was also given in Chen and Zhao (2020).

The above works have derived significant results of the impacts of communication delays on the distributed algorithms for ED in the SG. Notably, in practical power systems, especially in large-scale ones with distributed properties, the communication time delays are usually time-varying. Thus, establishing an effective distributed continuous-time algorithm with time-varying communication time delay is of great significance. Huang et al. (2019) designed a fully DED algorithm under strongly connected digraphs, incorporated the time-varying delays and estimated active power variables into the algorithm. Sufficient conditions in the form of linear matrix inequalities regarding the upper bound and derivatives of delays were given to guarantee the convergence of the algorithm to the global optimal solution. Yu M et al. (2020) studied the effects of random delay on the distributed consensus-based algorithms in solving EDP. To guarantee the convergence of the algorithm under the generation constraints, a random variable was introduced in Yu M et al. (2020) to model the random delay, and theoretical upper bounds of the delays were derived.

Another critical factor that appeared in communication networks is communication uncertainty. Note that communication uncertainties are ubiquitous in practical communication networks, including the uncertainties caused by fading channel, transmission losses, and data packet dropout. Meanwhile, some cyber-physical malicious attacks, such as edge manipulation attacks, could be modeled as specific types of uncertainties. Note that consensus-based ED algorithms often use linearly weighted discrepancy between the neighbor generators; the appearance of communication uncertainties will directly tamper the prescribed weights, and thus may alter the dynamics or even lead to divergence of algorithms. Hence, it is worth further exploring robust ED algorithms to suppress the adverse effects of communication uncertainties critically. Wen et al. (2018) suggested an adaptive consensus-based framework to deal with EDP in the presence of communication uncertainties. The proposed distributed continuous-time ED algorithm with the adaptive weight-adjustment technique is as follows:

$$\begin{cases} \dot{\zeta}_i(t) = 2c\gamma_i \sum_{j\in\mathcal{N}_i} [a_{ij}(t) + \omega_{ij}(t)][\zeta_j(t) - \zeta_i(t)], \\ \dot{a}_{ij}(t) = h_{ij}[\zeta_j(t) - \zeta_i(t)]^2,\ j \in \mathcal{N}_i, \end{cases} \tag{12}$$

where $\zeta_i(t)$ denotes the incremental cost of generator i, $h_{ij} = h_{ji}$ is the positive constant, and $\omega_{ij}(t) = \omega_{ji}(t)$ is the time-varying and unknown communication uncertainty. By designing the Lyapunov energy function, it was proved that the optimal dispatch of electric power could be assured against unknown communication uncertainties, and that the balance between power supply and demand could be met during the dispatch process.

5 Conclusions and future research directions

We have reviewed some research advances on distributed economic dispatch (DED) in the smart grid (SG). Notably, many interesting and important issues concerning how to design efficient DED algorithms for SG deserve further investigation. Some interesting and critical future research topics within this field are pointed out as follows:

1. Distributed security ED of the SG

As an important next-generation electricity infrastructure, the SG aims to yield smarter features than the traditional electric grid in multiple aspects by highly incorporating sophisticated communication and information technologies. Several cybersecurity challenges emerge within this context, especially related to connectivity of the underlying communication networks, normal functioning of power facilities, and privacy protection for data collection. At present, designing resilient ED algorithms for the SG subject to cyber-attacks on generators and/or on the underlying communication networks is essential, and thus deserves further investigation. Another critical research topic in the context of distributed security ED in the SG is how to design distributed privacy-preserving ED strategies for multiple generators.

2. Distributed fast ED in the SG with practical constraints

In the past few years, various fast coordination algorithms for multi-agent networks have been designed (Hu B et al., 2019; Hu HX et al., 2019). Serval distributed fast algorithms have been constructed and used to address the finite/fixed-time EDPs of the SG in the absence of practical constraints (e.g., the output constraints of distributed generators and the constraints over a distribution network). Practically, it is highly desirable to develop distributed fast ED algorithms such that the dispatch objectives can be achieved in finite or fixed time while satisfying the practical constraints.

3. Efficient initialization-free DED in the SG

Power balance is a prerequisite for successfully executing most of the existing ED algorithms in the SG. Some initialization-free DED algorithms have been reported. Nevertheless, most of the initialization-free DED algorithms have only asymptotic convergence speeds. How to construct more efficient initialization-free DED algorithms for the SG is still an outstanding issue and deserves further studies.

4. DED in the SG in the presence of smart energy storage batteries and flexible loads

It is not difficult to foresee that different kinds of smart energy storage batteries and flexible loads will be heavily integrated into the future SG, which will be essentially controllable resources to balance the power supply and demand. The developments of smart battery energy storage technology and scheduling technology of flexible loads will be helpful in peak shaving of power energy demands. Therefore, it will be an interesting topic to consider how to take full advantage of flexible loads and smart energy storage batteries in solving distributed EDPs.

5. DED in the SG with artificial intelligence technologies

There have been advances in artificial intelligence (AI) technologies for addressing various modern industrial problems. AI technologies and tools excel at dealing with data analysis, evolution behavior analysis, and prediction of engineering systems, system control, and complex decision-making problems. AI technologies (e.g., the deep learning approach, brain-like technology, and reinforcement learning approach) will provide new paradigms for dealing with DED in the SG, especially under uncertain environments.

Contributors

Guanghui WEN drafted the manuscript. Xinghuo YU and Zhiwei LIU helped organize the manuscript. Guanghui WEN revised and finalized the paper.

Compliance with ethics guidelines

Guanghui WEN, Xinghuo YU, and Zhiwei LIU declare that they have no conflict of interest.

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